A PLAIN
ACCOUNT
OF THE
Old and New STILES.

IN WHICH,

Their Difference is deduced from
clear PRINCIPLES;

AND

The Difficulties attending a perpetual Reformation of the CALENDAR, are
ſhortly Stated and Reſolved.

In a LETTER to a Member of Parliament.

LONDON:
Printed for A. MILLER, oppoſite *Catherine-Street*,
in the *Strand*.
M,DCC,LI.

[Price 1s. 6d.

1258

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BRITISH MUSEUM

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A
LETTER
Concerning the
Old and *New* STILES.

SIR,

ABOUT a Year ago you wrote me, that ſome Perſons of diſtinguiſhed Rank and Character, intended to bring into Parliament a Bill for altering the *Stile*, and conforming in that reſpect with the Uſage of our Neighbours on the Continent; deſiring that I would give you my Thoughts on the Subject, and explain the Difference between the *Old* and the *New* Stile, in a familiar, intelligible Manner; to ſave you the Time and Trouble of turning over the Authors that have treated of this Matter.

Now, I hear, ſuch a Bill is actually depending in the Houſe of Lords, and is likely to paſs into a Law; which muſt recall the Subject to your Thoughts, and perhaps make you wiſh I had been a little fuller, and more particular, in my Anſwer. I have therefore reviſed, and ſomewhat enlarged what I ſent you laſt Winter; and have added what was wanting to give you a clear View, not only of the *Julian* and *Gregorian* Stiles, but of the Rules that have been invented, from time to time, to adjuſt the *Lunar* to the *Solar* Year, and thence find the *Paſchal Limits*, as they are called, with the *Sunday* on which the *great Chriſtian Feſtival* ought to be celebrated; on which all the other moveable Feaſts depend.

In order to this, I aſſume the *Tropical* Year, and likewiſe the *mean Synodical Month*, or the Time between a New Moon and the New Moon next following, to be of the Quantities they are now fix'd at by the exact Obſervations of Aſtronomers.

From the Exceſs, or fractional Part, by which a Solar Year exceeds its greateſt Integer Number of Days, *i. e.* 365, the *Julian* and *Gregorian* Intercalations are eaſily accounted for. And from the Difference by which the whole Number of twelve *Lunations*, or *Synodical* Months, falls ſhort of the Solar Year, a Method of intercalating or inſerting the Months that reſult from the Repetition of this Difference is deduced ; ſo as after a certain *Cycle*, or Integer Number of Years, or Lunations, to bring back the *Conjunction*, or *New Moon*, nearly to the ſame Place where it had happened at the Beginning of the Cycle.

Theſe Things being once obtained according to the exacteſt modern Theory, it will be eaſy to examine and compare the ancient Forms of the Year ; to ſhew the Imperfection of the ſeveral Cycles, and to account for the Corrections they neceſſarily underwent from Age to Age : Of which ſome of the moſt remarkable Inſtances ſhall be given.

But firſt, I muſt beg Leave to premiſe a few Aſtronomical Principles, without which all that is to follow would be but imperfectly underſtood : Nor need you be alarmed at the Mention of *Aſtronomy* ; for all that we ſhall have Occaſion for at preſent, is ſo ſimple, as ſcarcely to need either Diagram or Inſtrument ; only a little Attention, and the knowing a few *technical* Words, which are to be had from a common Dictionary, or from the Definitions of ſome Treatiſe on the Uſe of the Globes.

Or if thoſe Principles are ſtill freſh enough in your Memory, you may paſs them over, and proceed immediately to the ſecond Section.

SECTION I.

Aſtronomical Principles and Definitions.

1. THE firſt Appearance, or *Phænomenon*, in Aſtronomy, is the diurnal Revolution of the Heavens, and all their Hoſt, from Eaſt to Weſt; cauſing the alternate Succeſſion of *Day* and *Night*, the *riſing*, *culminating* (or *ſouthing*) and *ſetting* of the *Sun*, *Moon*, and *Stars*. This Motion of the Heavens was, according to the *Ptolemaic Syſtem*, thought to be *real*; but later Diſcoveries and Reaſonings have ſhewn it to be *apparent* only, cauſed by the Rotation of the Earth on its Axis, the contrary Way; that is, from Weſt to Eaſt: which, to a Spectator, placed any where on the Earth's Surface, will exhibit pre-

ciſely the ſame *Phænomena*, and in the ſame Order, as the Motion of the Heavens would do in the *Ptolemaic* Hypotheſis.

The Reaſon of this is not difficult to apprehend; for the Poſition of all the heavenly Bodies being referred to the *Horizon*, that is to the Plane on which the Spectator ſtands, continued to the Heavens; it is all one whether they *riſe*, *culminate*, and *ſet*, by the Motion of the Heavens round a fixed *Axis*; or the *Horizon* goes to meet the heavenly Bodies, by a contrary Motion round the ſame *Axis*.

2. This Rotation of the Earth round its Axis is perfectly *equable*; at leaſt there has never been any reaſonable Suſpicion that it was not. Mark the Time when any *fixed Star* is preciſely in the *Meridian*, or to the South of any Place, for ſeveral Days ſucceſſively; and the Times between the *Southings* meaſured by a good Pendulum Clock will be preciſely equal: Or taking the Difference of the Times of ſouthing of any two fixed Stars, they will likewiſe be for ever equal: And indeed without this Equability of the Earth's Rotation, we could have no certain Meaſure of *Time* at all.

3. The *fixed* Stars were uſed for the Purpoſe of the foregoing Paragraph, becauſe they keep conſtantly the ſame Poſition with reſpect to each other; and thence derive their *Name:* whereas the *Planets* (or *Wanderers*) beſides the general apparent Motion of the Heavens, have each a particular Motion of his own: which to ſettle and compute makes the chief of Buſineſs of an Aſtronomer; but the Motions of the Sun and Moon are all that we are concerned with at preſent.

That the Sun has ſuch a particular Motion muſt have been very ſoon obſerved; becauſe at different Seaſons of the Year he riſes and ſets at different, and very diſtant, Places of the *Horizon*, and in the *Meridian* is elevated to different *Altitudes*; The Days (except under the Equator) are for half of the Year longer than the Nights, and for the other half ſhorter. None of which Varieties happen in the Phænomena of a *fixed* Star; which always riſes and ſets on the ſame *Points* (if it riſes and ſets) is, elevated to the ſame *Meridional Altitude*, and is ſeen the ſame time above the *Horizon* of any given Place.

It would be no leſs plain to an attentive Obſerver, that this apparent Motion of the Sun, among the fixed Stars, was from Weſt to Eaſt; becauſe a fixed Star, which is now viſible in the Weſt a little after Sun-ſetting, a few Days hence will be no longer ſeen; and a few Weeks after will appear in the Morning, to the Weſtward of the Sun.

4. The circular Path, which the Centre of the Sun deſcribes, while he is thus continually ſhifting his Place among the *fixed* Stars, is called the *Ecliptic*; becauſe whenever an Eclipſe happens, the Centres of the Sun and Moon muſt be in the Plane of this Circle, or not far from it.

It is divided into twelve equal Portions, or *Signs*; each of 30 *Degrees*, or 360th Parts of the whole Circumference: which are denominated from the principal Conſtellations through which the Circle paſſes. Their Names and Marks are as follows.

Sunt Aries, Taurus, Gemini, Cancer, Leo, Virgo;
♈ ♉ ♊ ♋ ♌ ♍
Libraque, Scorpius, Arcitenens, Caper, Amphora, Piſces.
♎ ♏ ♐ ♑ ♒ ♓

5. As from the equable Rotation of the Earth on its Axis (§ 2.) we obtained the fix'd Meaſure of a *Sidereal Day*; ſo from the annual Motion of the Earth round the Sun, or the *apparent* Motion of the Sun among the fixed Stars, we have now got the Meaſure of a *Sidereal Year*, *viz.* the Time in which the *Earth*, or *apparently* the *Sun*, deſcribes a whole Circle in the *Starry* Sphere.

6. Hence, likewiſe, we have got a *fixed* Plane, or *Baſis*, to which the Poſition of the *Orbits* of the heavenly Bodies, the Circles of the *Sphere*, and all the Phænomena of Aſtronomy muſt be referred; for in all Ages the *Ecliptic* Circle has been obſerved to paſs through the ſame *fixed* Stars; and that its Poſition muſt for ever continue the ſame, is demonſtrable from the phyſical Laws of Motion *, unleſs ſome foreign Action or Appulſe, as of a *Comet*, ſhould diſturb the Earth's Courſe.

7. Chiefly we are to obſerve, that the Earth's Axis, round which the diurnal Rotation is performed, is inclined to the Plane of the Ecliptic in an Angle of nearly 66 Degrees and a half; and conſequently the *Terreſtrial Equator*, to which

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* Newt. Princip. *Lib.* 1. *Prop.* 1.

that Axis is perpendicular, will be inclined to the Plane of the *Ecliptic* in an Angle which is the Complement of the former, to 90° that is, in an Angle of about 23°$\frac{1}{2}$.

8. From the Quantity of this Inclination, the Viciſſitude of the *Seaſons*, and the lengthening and ſhortening of the Days in different Climates, may be eaſily accounted for, according to the *Pythagorean* or *Copernican* Hypotheſis; if we add only this *Lemma*, " That if a globular Body is " expoſed to a diſtant luminous Body, as the " *Sun*, exactly one half of it will be illuminated, " and the other not; and that the Termination of " Light and Shade will be a Circle."

Draw, with a String and a Bit of Chalk, a large Circle on the Floor of your Room, to repreſent the *Ecliptic*; and by drawing two Diameters, divide it into four equal Parts, AB, BC, CD, DA, reckoned from the right to the left, as you ſtand within the Circle; ſo ſhall A, B, C, D, repreſent the *cardinal Points* of the *Ecliptic*, A and C being the *Equinoctial*, and B and D the *Solſtitial* Points. Place, with its Centre over the Point A, the ſmalleſt terreſtrial Globe you can procure, ſo as the *North* and *South* Points of the *wooden Horizon* may lye in the Direction DB. Then having

raiſed

raiſed the Globe till its Axis make an Angle of 66° ½ with the *Horizon*, or the Floor, and lighted a Taper in the Centre, at the Heigth of the wooden Horizon above the Floor, to repreſent the Sun ; you will find that the Termination of Light and Darkneſs on the Globe will paſs through either *Pole*, and will divide the Equator, and all its Parallels, equally ; and moving the Globe equably on its Axis from Weſt to Eaſt, to repreſent the diurnal Rotation, any Place of its Surface will have equal Day and Night. This is the *Vernal Equinox*.

Remove now the Globe to the Point B, keeping the *North* and *South* Points of the *Horizon*, and the Elevation of the *Axis* the ſame as before ; then will the higheſt Point of the illuminated Hemiſphere reach to 23 ½°, beyond the *North* (or elevated) Pole ; and its loweſt Point will fall as much ſhort of reaching the *South Pole* : and therefore in this Poſition of the Earth, the diurnal Rotation can deprive no Place within the North *Polar Circle*, that is, no Place within leſs than 23 ½° of the Pole, of the Sun's Light : As within the South *Polar Circle*, the ſame Rotation cannot bring it out of the dark Hemiſphere.

In intermediate Places, *i, e.* from the *Polar Circles* to the *Equator*, it will appear, that, in the Northern Hemiſphere, the enlightned part of the *Parallel* of any Place, exceeds the unenlightned; and ſo much the more as the Place is nearer to the Polar Circle; and the contrary happens in the *Southern* Hemiſphere: that is to ſay, at the Summer *Solſtice*, the greater the *North-Latitude* of ſuch a Place is, the longer is the longeſt Day; and the greater the *South-Latitude* is, the ſhorter is their ſhorteſt Day.

The Globe being next placed at C, the Phænomena of the *Autumnal* Equinox, equal Days and Nights, return.

Laſt of all, when it is placed at D, the loweſt Point of the Illuminated Hemiſphere reaches 23½° beyond the *South* Pole; and it's higheſt Point wants the ſame Diſtance of reaching the *North* Pole: which exhibits again all the *Solſtitial* Phænomena, but of a contrary Denomination, Day for Night, and Night for Day.

And when it is placed at any other Point of the *Ecliptic*, as between A and B, B and C, &c. the Phænomena of the correſponding Seaſons, with Reſpect to the Lengthning or Shortening of the

Days, will be explained with the ſame Facility; and the Reaſon of the Arithmetical Rules for computing them will be ſeen.

If you have not a Globe at Hand, it may be ſupplied by a ſmall wooden Ball, moveable on a a Piece of Wire; but on which you have firſt deſcribed with Ink the Equator and the Polar Circles. But indeed, after what has been ſaid, it is hoped the Imagination of any attentive Perſon will need no *Apparatus* of Inſtruments to help him to a clear Notion of this Matter.

9. The Equinoxes happen when (as in the laſt §.) the Earth is at A and C; that is, when the common Interſection of the Planes of the *Ecliptic* and *Equator* paſſes thro' the Centre of the *Sun*. And if, as was there ſuppoſed, the Axis of the Earth kept conſtantly it's *Paralleliſm*; this Interſection would remain for ever directed to the ſame Point of the Heavens: and the *Tropical* Year, or *the Time which the Sun takes to move from a Cardinal Point of the Ecliptic, the Vernal Equinox for Inſtance, 'till he returns to it again*, would be the ſame as the *Sidereal* Year, deſcribed in § 5.

But it hath been found by Obſervation, That the Interſection of thoſe Planes has a Motion from Eaſt to Weſt; which however ſlow becomes

comes, with its Effects, very ſenſible in a Number of Years. And hence the *Tropical* Year is ſhorter than the *Sidereal*, by about 20° Minutes of an Hour. If laſt Year, for Inſtance, the Vernal Equinox happened in the Point P of the *Ecliptic*, where the Interſection of the Planes paſt thro' the *Sun*'s Centre, it will happen this Year before the Sun has reached the Point P, becauſe the Interſection has moved Weſtward to meet the Sun 50 Seconds of a Degree. Say, therefore by the Rule of Three, if 360°, the whole Circle, give $365\frac{1}{4}$ Days, or a Year, what will 50″ or $\frac{1}{72}$ of a Degree give? And the Anſwer will be a little more than 20 Minutes of an Hour.

Hence likewiſe, the Conſtellations thro' which the Ecliptic paſſes, and from which it's Diviſions or *Signs* took their Names, are conſiderably diſplaced with reſpect to the Cardinal Points; which have left them far to the Eaſt of their firſt Situation, tho' the ſame Names are ſtill retained. And the Quantity of this Diſplacement, from the Time that *Eudoxus* modelled the Sphere *, furniſhed to Sir *Iſaac Newton* a Means of correcting the *Chronology* of Ancient Kingdoms: as may be ſeen in his Treatiſe on that Subject, or in Mr. *Reid*'s Abridgment of it.

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* Before Chriſt, about 430.

The ſame great Author gives the Phyſical Cauſe of this Motion, or *Preceſſion*, as it is called, of the Equinoxes, in his Principles of Natural Philoſophy. See likewiſe the Commentaries of Dr. *Pemberton*, and Mr. *Mac Laurin*.

But to give a clear and familiar Idea of the Motion itſelf; the Plane of a Table repreſenting the Ecliptic, we need only paſs a Pin perpendicularly through the Center of a Circular Piece of Paſte-board or Card-paper, and lay it on the Table, ſo as the Circle may repreſent the Plane of the Equator. For then preſſing gently the Pin's Point into the Table, that it ſlide not, if we move the Circle round from left to right, a Tangent Line to that Point of it's Circumference in which it touches the Table, or a Diameter of the Circle, parallel to it, will every where repreſent the Poſition of the Interſection of the Planes of the Ecliptic and Equator, while the Axis deſcribes a Cone, whoſe Angle at the Vertex, or Point of the Pin, is twice $23\frac{1}{2}°$, or 47°.

Now the Motion annually being about 50″, or $\frac{1}{72}$ of a Degree, to go round the whole Ecliptic or 360°, will require 72 Times 360, or 25920 Years; that is, after 25920 Years, the Vernal Equinox will happen in the ſame Point of the

Heavens as it does now; and in half that Time, *viz.* 12960 Years, the Axis of the Earth will point to a fix'd Star 23½° on the other Side of the Pole of the *Ecliptic*, that is 47° diſtant from the Star it now Points to.

10. Thus much for the Solar Year: proceed we now ſhortly to explain the *Inequality* of *Natural* Days; and to define the *mean* Aſtronomical Day, ſuch as a well going Clock meaſures.

A *Natural* Day * is the Time from Noon to Noon; that is, from the Sun's Centre being in the Meridian to Day, and it's returning to it the Day following.

11. That this Day is longer than a *Sidereal* Day, deſcribed in § 2, is evident. For ſuppoſe, To-day, the Centre of the Sun and a fix'd Star to come to the Meridian together; To-morrow the Sun by his apparent Motion in the Ecliptic, will have got ſome way to the Eaſt of the Star, and conſequently his Centre muſt paſs the Meridian later.

12. Yet is not the Natural Day always of the ſame Length; becauſe the Ecliptic in which the

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* This is by ſome Writers called a Civil Day, and the time of Sun-ſhine a Natural Day.

Sun performs his annual Courſe; and the Equinoctial, from the Equable Revolution of which, Time is meaſured, interſect each other in an Angle of 23½°. Whence it happens that the Difference between the Solar and equal Sidereal Day, (deſcribed in the laſt §) is ſometimes greater and ſometimes leſſer. Imagine for Inſtance, that the Sun ſets out from the Vernal Equinoctial Point, preciſely at Noon, and that in the ſpace of a Day he has deſcribed a Degree of the Ecliptic. Thro' the *Sun*'s Centre and the Poles of the World, let a great Circle of the Sphere paſs; and it will be plain, that, on Account of the aforeſaid Obliquity, this Circle, which cuts off a Degree of the Ecliptic, will cut off leſs than a Degree of the Equinoctial, in the Ratio, nearly of the Baſe of a Right-angled Triangle, one of whoſe acute Angles is 23½°, to its Hypothenuſe. But now the Arc of the Equator thus cut off, which is likewiſe called the Sun's *Right Aſcenſion*; is the very Meaſure of the Difference (in the laſt §) between the *natural* Day that follows the vernal Equinox, and the invariable Sidereal Day.

13. Let us next ſuppoſe the Sun to ſet out from one of the *Solſtitial* Points, as from the beginning of *Cancer*, and to move in the *Ecliptic* one Degree in a Day. Then I ſay, the two Circles thro'

thro' the Poles of the Sphere, between which this Degree lies, will intercept more than a Degree of the *Equinoctial*; becauſe the *Sun*'s apparent Diurnal Motion being now in a *leſſer* Circle, *viz.* the *Tropic* of *Cancer*, a Degree of a great Circle, *viz.* the *Ecliptic* (which here touches, and nearly coincides with it) will take up more than a Degree of that *leſſer* Circle; and conſequently the Circles of Declination will intercept more than a Degree of the Equinoctial: that is, the *Aſcenſional Difference*, belonging to the Day next after the Solſtice, which meaſures the Difference of that Day and of the Sidereal Day, will now be greater than a Degree *.

In the former §, at the Equinoxes, it was leſs than a Degree; and in ſome intermediate Point of the *Ecliptic*, it will be preciſely equal to it: that is, it will be of its *mean* Magnitude, the ſame as if the *Ecliptic* and *Equinoctial* were coincident.

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* We may, as a farther Illuſtration, uſe the following eaſy Examples. If a Room is 12 Foot wide, and I walk, not directly acroſs it, but from one Corner to the other; when I have advanced 12 Feet, I ſhall not for that have reached the other Side of the Room. Again, if one end of the Room is wider than the other, and the Boards of the Floor taper in the ſame Proportion, as they ſeem to do in a Piece of Perſpective; then the Line which meaſures the Breadth of 12 Boards at the narrow End, muſt meaſure the Breadth of a leſſer Number at the other End of the Room. The Application of both Examples is obvious.

14. But

14. But altho' theſe Circles were one and the ſame, the natural Days would not be perfectly equal, on another Account. Which is, that the *Earth*'s Orbit is not circular, but an *Ellipſis* or Oval, in one of whoſe *Focus*'s the Sun is placed. And it being a Law of Nature, that a Planet, moving round a Central Body, deſcribes equal *Areas* in equal Times; in Winter, when the Earth is nearer the *Sun*, to compenſate the ſhortneſs of the deſcribing Ray, her Angular Velocity is greater; as in the Summer half Year, to compenſate the length of the Ray, preſerving the equable Deſcription of the Area in proportion to the Time, the Angular Velocity is leſſer; this of conſequence increaſes and diminiſhes the Sun's apparent Velocity in the Ecliptic; and complicated with the other Cauſe of Inequality, produces the Differences that are mark'd in an *Equation Table*, of the *mean* or *equal* Time mark'd by a good Clock, and the *apparent* Time on a Sun-Dial.

15. We may now eaſily conceive what a *mean* or *equal* Day is. When the *Sun* ſets out from any Cardinal Point, as the beginning of *Aries*, to run his annual Courſe in the *Ecliptic*; let us, with Dr. *Gregory*, ſuppoſe that another (fictitious) *Sun* ſets out from the ſame Point in the *Equinoctial*,

with a Motion perfectly equable, and ſo adjuſted, that the two *Suns* ſhall return to the Equinoctial Point at the ſame Inſtant. Then from the two preceeding § §, it is plain the *fictitious* Sun will be in the Meridian ſometimes later and ſometimes ſooner than the true *Sun*: and four Times in the Year they will be in the Meridian together: but the Days of the *fictitious* Sun will be all perfectly equal. And of ſuch Days, and their Subdiviſion, we ſpeak when we ſay a Year, a Month, or a Cycle, conſiſts of ſo many Days, Hours, Minutes, &c.

* * *

SECT II.

Of the Correction of the Calendar by Julius Cæſar.

16. IT follows from what has been ſaid, that the beginning of the Year, the *Caput Anni*, which Nature itſelf points out, is, when the Sun is in one of the Cardinal Points of the Ecliptic; making the Days and Nights equal, or of their greateſt or leaſt Quantity. And accordingly we find that from one or other of thoſe

Points, the Solar Year has been generally reckoned in all civilized Nations.

Its Diviſion, likewiſe, into the four Quarters, of *Spring*, *Summer*, *Autumn*, and *Winter*, is naturally indicated by the Courſe of the Sun; and, to mark the Seaſons proper for the ſeveral Works of *Agriculture*, *Planting*, *Navigation*, &c. the ancient Writers uſe the *heliacal Riſing* or *Setting* of ſome remarkable Star; ſuch as the *Pleiades*, the *Dog-Star*, &c. that is, the Time when a certain Star, emerging from the Sun's Rays, becomes viſible in the Morning before Sun-riſing, or ceaſes to be viſible after he ſets; the ſame Phænomenon as was deſcribed in § 3.

This would anſwer the intended Purpoſe well enough, for ſeveral Ages; but, at length, the Præceſſion of the Equinoxes, deſcribed in § 9, would introduce a ſenſible Error: making the Seaſon which formerly anſwered to the *heliacal* Riſing or Setting of any particular fixed Star, to fall now conſiderably earlier.

To remedy this Inconvenience, as well as to furniſh the minuter Subdiviſions of Time, which the Dates of Events, Tranſactions, and aſtronomical Appearances require, it was neceſſary to invent a *Calendar*, that is, a Diviſion and Subdiviſion of the Year into Months and Days; ſo contrived, that the Beginning of the Year, and every Day of it, ſhould, nearly enough for civil Uſe, that is, within leſs than 24 Hours, anſwer to the ſame Seaſon, or to the ſame Degree of the *Ecliptic*. Nor need we be ſurprized, if the firſt Eſſays towards this were ſo rude and imperfect, as from ancient Hiſtory they appear to have been; or that even the *Julian* Year needed the Correction of *Gregory* XIII. when we conſider how difficult it ſtill is to determine the exact Length of a *tropical* Year; though Inſtruments for Obſervation have been brought to almoſt all the Perfection of which they ſeem capable.

18. But to explain this Matter intelligibly, let us firſt ſuppoſe the *tropical* Year, that is, the Time from the Sun's leaving a cardinal Point of the Ecliptic, till he returns to it, to conſiſt of an integer Number of Days, as 365; then it is plain, that the Repetition of 365 Days continually, would reſtore the Beginning of the Year to the ſame Point of the Ecliptic; and the Form of the Year, perfect in itſelf, would need no Intercalation, no Addition or Subtraction of Days.

Let us next ſuppoſe the Year, which really conſiſted of 365 Days, to have been, by Miſtake, made to conſiſt of 366, a Day too much; and let the vernal Equinox, or Beginning of the Year, be *March* 10th, at Noon: in the *true* Calendar, next Year the vernal Equinox will happen after 365 Days; but the *reputed* Year of 366 Days will not end till *March* 11; the ſecond Year it will not end till *March* 12, and ſo on; gaining a Day of the true Calendar every *tropical* Year; till having gone through all the Seaſons, *Spring*, *Summer*, *Autumn*, and *Winter*, after 365 *reputed*, or 366 *tropical* Years, the Beginning of the Year ſhould return to its firſt Place in the *true* Calendar: While, in the mean time, the Equinox would move backward, with reſpect to the *falſe* Calendar, through *February*, *January*, *December*, &c.

The like would happen, but in a contrary Order, if the Year were, by Miſtake, rated at 364 Days, a Day too little. For in that Caſe, the Beginning of the reputed Year would move backwards, at the ſame rate, from the cardinal Point; and the cardinal Point would move forward from the Days marked in the falſe Calendar, till after 364 *tropical* Years, or 365 *reputed*

Years, Things were reſtored to their firſt Situation.

19. From what has been ſaid in the laſt Paragraph, it is plain, that accordingly as the Year is *over-rated*, with reſpect to its Length, or *under-rated*, the cardinal Points, that is, the Equinoxes and Solſtices, will go *backwards* or *forwards* in the Calendar; and that the greater the Difference between the Year of the Calendar and the true *tropical* Year is, ſo much the ſooner will the Beginning of the Year return to its firſt Place, with reſpect to a true Calendar; and, on the contrary, the leſſer that Difference is, ſo much the longer will the Time of its Revolution be.

Let us take, for example, the Year of 360 Days, ſaid to have been in uſe among the *Egyptians*, *Chaldeans* and *Perſians*, in the Infancy of Aſtronomy. And this falling ſhort of the tropical Year, by near five Days and a quarter, which, in 70 Years, will amount to a Year of 365 Days, the Beginning of that ancient Year would wander back through all the Seaſons, in the Space of one Man's Life; to prevent which, an Intercalation of 5 Days was made at the End of every Year.

But the Number of 365 Days ſtill falling ſhort of the tropical Year, by near a quarter of a Day, and this Difference amounting, in four times 365, or 1460 Years, to a whole Year; in that Period the Beginning of the Year would go backwards, till it returned to its firſt Situation with reſpect to the Seaſons: that is, it would go backwards, ſlower than before, nearly in the Ratio of 6 Hours to 5¼ Days, or of 1 to 21.

20. At laſt, *Julius Cæſar* obſerving what Confuſion had crept into the *Roman* Calendar, partly through the Uſe of a *lunar* Year, partly through the Ignorance, Corruption, or Careleſſneſs, of the College of Prieſts, whoſe Buſineſs it was to make the Intercalations; had Recourſe to the Mathematicians of *Alexandria*; and by the Help of *Soſigenes* particularly, whom he ſent for on purpoſe, ſettled and eſtabliſhed, by his Edict, as perpetual Dictator and ſoverein Pontiff, the Form of the Year, which ſtill bears his Name.

His Correction of the Year conſiſts ſolely in this; that the 6 Hours by which the tropical Year exceeds 365 Days, are now taken into the Account; and the Day which they make up in four Years is intercalated, either as anciently, by calling two Days by the ſame Name, *viz. February* 24, and the Day following it, *ſexto Calendas Martii*, (whence the Name of a *biſſextile* Year) or, by giving to the *February* of every fourth Year 29 Days, inſtead of 28; and conſequently to the whole Year 366 Days, which we call a *Leap Year*.

For inſtance, if the Equinox of any Year, ſhould happen *March* 11, at Noon, the Year after it will happen at Six in the Evening of the ſame Day, the ſecond at Midnight, the third at Six in the Morning *March* 12, and the fourth at Noon of the ſame Day, if no Day was intercalated.

But if *February* 24 is twice reckoned, the Day which ſhould have been *March* 1, is this Year called *February* 28; then what ſhould have been *March* 2, will be *March* 1; and *March* 12 will be *March* 11. So that the Equinox will anſwer to the ſame Day of the Month, and to the ſame Hour, as four Years before.

This Rule, however ſimple, was at firſt miſunderſtood and miſapplied, by intercalating every third Year inſtead of every fourth; by which means, 24 Hours being reckoned every three Years, inſtead of 18 Hours, there was an Over-reckoning of 6 Hours, which in 12 Years would amount to a Day, and in 36 Years to 3 Days. But this Miſtake was rectified by *Auguſtus*, and the Calendar reſtored according to *Cæſar*'s Intention, as if no Error had been committed.

SECTION III.

Of the Gregorian *Correction of the Year.*

21. IF the *Tropical* Year was preciſely $365\frac{1}{4}$ Days, the *Julian* Stile had needed no Correction; but its Length, by the beſt Obſervations, is found to be 365 Days, 5 Hours, 48′. 55″, leſs than what the *Egyptian* Aſtronomers aſſumed it, by 11′ 5″, which in 4 Years riſes to $44\frac{1}{3}$ Minutes. Seeing therefore the Year (in § 20.) was 11′ 5″ greater than its *true* Length, the cardinal Points will, by § 19, go backward by ſo much every Year; and the Equinox, which we juſt found was, by intercalating a Day, to return to *March* 11, at Noon, every fourth Year, will now really happen $44'\frac{1}{3}$ ſooner; that is, at 11h. 15′. 20″. Say therefore, "If a Difference of "$44'\frac{1}{3}$ give 4 Years, what will 1440 Minutes, "or a whole Day give?" The Anſwer will be very near 130 Years.

Whence it will follow; that as formerly when we took the Year ſhort of its (*ſuppoſed*) juſt Length, by 6 Hours, we were obliged, in order to keep the cardinal Points and the Days of the Calendar to their Places, to *add* a Day in 4 Years. So now having taken the Year 11′ 5″ longer than its juſt Quantity, we muſt *ſubtract* from the *Julian* Account one Day in 130 Years. Which is the very Rule that *Roger Bacon* is ſaid to have given for the ſame Purpoſe 500 Years ago, and which comes nearer the Truth than Pope *Gregory*'s.

22. This Anticipation of a Day in 130 Years had, in Pope *Gregory*'s Time, *anno* 1580, brought the vernal Equinox from the 21ſt of *March*, where it was in 325, when the Council of *Nice* publiſhed their Canon, back to the 11th of the ſame Month, the Difference being near 10 Days: For as 130 Years is to one Day, ſo is 1257 Years to 9. 7^{d}; cauſing an intolerable and growing Diſcordancy between the *aſtronomical* and the *eccleſiaſtical* Equinox.

To rectify which, and that the like might not happen for the future; in the Year 1581, the Pope iſſued his Bull; ordering, 1°. That, in 1582, the 5th of *October* ſhould be called the 15th, and ſo on; by which means, the 11th of *March*, when the Equinox then fell, would be changed into the 21ſt, its Place at the Time of the Council of *Nice*. 2°. The aſtronomical Tables and Obſervations of that Time making the Anticipation of the Equinox to be at the Rate of 1 Day in 133 or 134 Years, that is, of 3 Days in 400 Years; to prevent its Effects in Ages to come, he ordered, that every 400 Years 3 Days ſhould be ſuppreſſed, by reckoning the grand Jubilees, or centenary Years 1700, 1800, 1900, (which are not diviſible by 4, and) which, in the old Calendar, are Leap Years, to be common Years of 365 Days; and the Year 2000, and every centenary Year to come that is diviſible by 4, to be Leap Years: This Rule not taking place till 1700, becauſe the 10 Days already ſuppreſſed, exceeding the juſt Quantity, included the Correction due from 1582 to 1700.

23. This Correction of the ſolar Year, though not altogether perfect, becauſe the Number of Years, (130) in which a Day is to be ſuppreſſed, is not exactly the third Part of four Centuries, is nevertheleſs ſufficient for all Purpoſes, civil and eccleſiaſtical; keeping a cardinal Point of the Year to within a Day of the ſame Place of the Calendar, for more than 5000 Years.

24. I ſhall only add, as a Matter of ſome Curioſity, that although the Rule was abſolutely perfect, it is only *one* cardinal Point that can be forever brought back to the ſame Day of the Month; that is to ſay, ſuppoſing the vernal Equinox were, by the *Pope*'s Correction, or ſome other of that kind, kept always to the 21ſt of *March*, from Leap Year to Leap Year, and from Age to Age; the *autumnal Equinox*, and the two *Solſtices* would ſhift their Places in the Calendar with a ſlow, unequal Motion, and after an entire Revolution of the *Aphelion* of the Earth's Orbit, return to their former Places; becauſe the *Aphelion*, or the Place where the Earth is at her greateſt Diſtance from the Sun, is found to have a ſlow Motion, with reſpect to the fixed Stars. It is at preſent oppoſite to the Sign of *Cancer*, which makes the Time from the *vernal* to the *autumnal* Equinox to exceed the *Winter Half-Year* by ſome Days. But when hereafter the *Aphelion* comes to be in oppoſition to *Capricorn*, the reverſe will happen; and conſequently the *autumnal Equinox*, and the *Solſtices*, will fall on different Days from what they do at preſent.

SECTION IV.

Of the Cycle of the Sun; *or, more properly, the* Cycle of the Sunday, *or* Dominical Letters; *and its Interruption by the* Gregorian *Correction.*

25. A Common Year of 365 Days, containing 52 Weeks and one Day over, ſuch a Year will always begin and end on the ſame Day of the Week. If *January* 1 is *Sunday*, (*December* 30 compleating the 52 Weeks) *December* 31 will likewiſe be a *Sunday*, and the next Year will begin on *Monday*. For the ſame Reaſon, the ſecond Year will begin on *Tueſday*, the third on *Wedneſday*; and ſo on for ſeven Years; till *January* 1 of the eighth Year falling again on a *Sunday*, the like Order runs through all the following *Septenaries* of Years; ſuppoſing there was no Intercalation, or *Leap Year*.

Imagine the Days of every Week, beginning with the firſt Week of *January*, to be marked with the ſeven Letters A, B, C, D, E, F, G, reſpectively, and the ſame Marks to be repeated continually; if *January* 1, of the firſt Year, was a *Sunday*, marked with the Letter A, the ſame Letter would ſtand over-againſt every *Sunday*, to Perpetuity; the *Mondays* would be marked with B, the *Tueſdays* with C, *&c.* But *January* 1, of the ſecond Year, would be marked with B, *January* 1 of the third Year, by C, of the fourth, by D, *&c.* till the firſt Day of the eighth Year came back to A; and the ſame Order recurred in all the following Septenaries of Years.

But if, inſtead of marking always the ſame Days of the Week by the ſame Letters, and conſequently making the Letter of New-Year's-Day run through all the ſeven, in the Order A, B, C, D, E, F, G; we ſhould find it more convenient to give to *January* 1, every Year, the Letter A, then the *Sunday* Letter would run through all the ſeven, A, G, F, E, D, C, B, in a contrary Order.

For if A denotes *January* 1, being *Sunday*, *December* 31 will alſo be a *Sunday*, and marked with the ſame Letter; and *January* 1 of the next Year ſhould be B. But if we would have the Character of New-Year's-Day ſtill to be A, we muſt move the whole Syſtem of Letters one Place foreforward; ſo that A taking the Place of B, G may ſucceed to that of A, and thence become the *Sunday* Letter; F may be in the Place of G, and ſo ſtand for *Saturday*; and ſo on, through the ſecond Year: and, for the ſame Reaſon, in the third Year, F ſhall take the Place of G of the ſecond Year, or the Place of A of the firſt Year; and, in like manner, E, D, C, B become Dominical Letters ſucceſſively, till the Septenary is compleated, and the ſame Order begins again from A.

26. In this caſe, the *Cycle of the Sun* would be no more than 7 Years. But becauſe, in every Leap Year, a Day is inſerted between *February* and *March*, by the Name of *February* 29, if we would ſtill keep the Letter A affixed to *January* 1, the Dominical Letters which have ſerved for the Months of *January* and *February*, muſt, for the reſt of the Year, be changed into the next lower; G into F, F into E, E into D, *&c.*

For the Days from *January* 1 to *February* 28, both incluſive, being 59, that is, 8 Weeks and 3 Days, the Letter belonging to *February* 28 muſt be C. Suppoſe this likewiſe to be the *Dominical Letter*, or that *February* 28 is *Sunday*, then *March* 7, with the Letter C, is the next *Sunday*, in a common Year;

Year. But if *Feb.* 29 is intercalated, as in the Leap-years, the next *Sunday* will be *March* 6th, with the Letter B, which will therefore mark all the *Sundays* for the reſt of the Year. And the ſame Thing would hold whatever the Dominical Letter of *January* and *February* had been *.

27. Now the Changes of G into F, F into E, E into B, *&c.* being *ſeven*, according to the Number of the Letters, and *one* of theſe Changes, happening in four Years; the Number of Years, or *Cycle*, in which all the Varieties of the *Dominical Letter* can happen, till the ſame Dominical Letter, and the ſame Order is reſtored, will be 4 times 7, or 28 Years, *the Cycle of the Sun*: As in the following Scheme.

G, F 1	B, A 5	D, C 9	F, E 13	A, G 17	C, B 21	E, D 25
E 2	G 6	B 10	D 14	F 18	A 22	C 26
D 3	F 7	A 11	C 15	E 19	G 23	B 27
C 4	E 8	G 12	B 16	D 20	F 24	A 28

In which the firſt Year of the Cycle is aſſumed a *Leap-year*, with the Dominical Letters G F, and conſequently the Years that count one more than a multiply of 4, as 5, 9, 13, *&c.* are Leap-years, with double Letters.

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* The ſame Reaſoning will hold, if we mark *Feb.* 24 and 25, both with the ſame Letter F.

27. Now when theſe Letters, (in Imitation of the eight *Nundinal* Letters of the ancient *Romans* *) were firſt affixed to the Calendar, for the Uſe of the Chriſtian Church ; the Beginning of the *Cycle* happened to be ſo placed, that repeating the ſame Cycle backwards to the Birth of *Chriſt*, the firſt Year of our vulgar *Æra*, would be the 10th of the Cycle with the Letter B, its Head with the Letters G F falling 9 Years before. Whence to find the Year of the *Cycle* for any given Year of our Lord, we have this

RULE (for the *old Stile.*)

To the given Year add 9, *divide the Sum by* 28, *and* (the Quotient marking the Number of Revolutions of the Cycle) *the Remainder will be the Year ſought :* And the *Dominical Letter is had from the Table by Inſpection.* If there is no Remainder, the Cycle is compleated under the Letter A.

But if, from the Number of the *Cycle*, you would find the *Dominical Letter* without a Table, *add to the Remainder, or Year of the Cycle, as many Units as it contains of* 4*'s* (or Leap-years) *divide*

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* See *Blondell,* Hiſtoire du Calendrier : tho' others are of a different Opinion.

vide the Sum by 7, *and, neglecting the Quotient, the Remainder will ſhew the Dominical Letter,* reckoning as below.

0, 1, 2, 3, 4, 5, 6.

A, G, F, E, D, C, B.

And if it is a Leap-year, the Letter found will be that which ſerves at the Beginning of the Year.

EXAMPLE I.

To find the Cycle of the Sun, and Dominical Letter for the Year 1743.

Adding 9, 'tis 1752, which divided by 28, quotes 62 (the Revolutions of the *Cycle* ſince, nine Years before the Birth of Chriſt) with the Remainder 16, the Cycle ſought, againſt which ſtands the *Dominical Letter* B; that is, in the *Julian* Calendar, all the Days marked with B, in the Year 1743, *viz. Jan.* 2, *Jan.* 9, *Jan.* 16, *&c.* were *Sundays.*

Or, if you would find the *Letter* by the Rule laſt given; To the Remainder 16 add 4 for the Leap-years, the Sum is 20, and this divided by 7,

leaves

7, leaves the Remainder 6, anſwering to the Letter B.

The Reaſon of adding Unity for every Leap-year is plain from the Table; where we ſee every Column takes up five Letters: And therefore to find what Letter ſtands in any aſſigned Place, we muſt reckon 5, inſtead of 4, in all the Columns between that Place, and the Beginning of the *Cycle*.

EXAMPLE II.

For the Year 1751, adding 9, 'tis 1760, which, divided by 28, leaves the Remainder 24 for the Number of the Cycle; add to this 6, for the Leap-years, aud the Sum 30 (divided by 7) leaves the Remainder 2, anſwering to the Dominical Letter F.

EXAMPLE III.

In like Manner the Cycle of the Sun for the Leap-year 1784, will be found to be 1; anſwering, we know, to the *Sunday* Letters, G F.

By this Rule ſome uſeful Queſtions may be reſolved; as, "To find what Day of the Week "a given Day of any Month of a certain Year "fell upon?" Or, having given the Day of the 1ſt, 2d, or 4th Week of any Month, to find the

"Day of the Month?" And for this Purpoſe the following *Memorial* Diſtich, or ſome other like it, may be uſed;

Aſtra Dabit Dominus, Gratiſque Beabit Egenos;
Gratia Chriſticolæ Feret Aurea Dona Fideli:

marking, by the initial Letters, the Letter in the Calendar for the firſt Day of each of the twelve Months in their Order, to ſave the Trouble of counting from *Jan.* 1.

To find, for Inſtance, on what Day of the Week *July* 24, 1751, will fall, *July* 1, being marked with G (by the Diſtich) and the *Sundays* in this Year with F (by the 2d Example above) the laſt of *June* is *Sunday*, and the 7th, 14th, 21ſt of *July* will be *Sundays* likewiſe, whence the 24th will be *Wedneſday*.

And, *vice verſa*, the *Sundays* being given, the Days of the Month on which the *Wedneſday*, *Thurſday*, &c. of any Week fall, will be eaſily found.

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SECTION V.

Of the Dominical Letter in the Gregorian *Calendar.*

THE Solar Cycle, according to the *Old Stile*, being ſo particularly explained, its Alteration in the *New*, the Omiſſion of 11 Days, will be eaſily underſtood.

28. Let us ſuppoſe, that 11 Days are to be ſuppreſſed in the Beginning of the Year 1752; that is to ſay, that immediatly after *Dec.* 31, 1751, we are to reckon *Jan.* 12, then F being the Dominical Letter of 1751, the ſeveral Days will in the *Julian* Calendar ſtand as in the Left Hand Column below: In which F, (*Dec.* 29) having been *Sunday*, *Jan.* 5th, 12th, *&c.* mark'd with E will be *Sundays* likewiſe. And if we place the Days of the ſame Month, with their Letters, as they ſtand in the Right Hand Column the 12th againſt the 1ſt, the 13th againſt the 2d, *&c.* then becauſe the *nominal* Change of the Days of

Decr. 29—F*
1751. 30—G
31—A

Jan. 1.—A—E—12 Jan.
2.—B—F—13
3.—C—G—14
1752. 4.—D—A—15
5.—E*-B—16
6.—F—C—17
7.—G—D—18
8.—A—E—19
9.—B—F—20
10.—C—G—21
11.—D—A—22
12.—E*-B—23
13.—F—C—24
14.—G—D—25
15.—A—E—26
16.—B—F—27
&c. &c.

of the Month alters not the Place of *Sunday*, or of any other Day of the Week; *Jan.* 16th, 23d, *&c.* will, in the *New Stile* be *Sundays*, with the Letter B. And the Conſtitution of the Calendar requiring that the Days of the Month be marked with the ſame Letters as before; *Jan.* 1, with A, *Jan.* 2d, with B, and ſo on; it is plain, that to find the Dominical Letters for the *New Stile*, we have only to change E*, F, G, A, B, C, D, into B*, C, D, E, F, G, A, reſpectively, which is done by *reckoning three Places backward, or four Places forward, from the Letters of the old Calendar*.

Thus having found the Dominical Letter for 1751, which is F, reckon backwards, E, D, C; ſo ſhall C be the Dominical Letter for the ſame Year in the *Gregorian Calendar*: And for 1752

E, D, being the Dominical Letters, in the *Old* Stile, B, A ſhall be theſe of the *New*.

Or thus,

Inſtead of counting three Places backwards, let us ſuppoſe, all the Cycles of 28 Years, marked with G F, E, D, *&c.* as in the Table, to begin 8 Years later; and conſequently that inſtead of 9 we add only Unity to the Year of our Lord. Then having by this Means brought the current Cycle (and all that preceded it) 8 Years lower, and the two Leap-years in theſe 8 Years, employing 2 Letters, each: the Letters G, F, E, D, *&c.* will be each removed ten Places lower than where they formerly ſtood; that is a whole Round of the 7 Letters, and 3 over; as below.

1,	2,	3,	4,	5,	6,	0,	1,	2,	3,	4,	5,	6,	0,	1,	2,	3,	4,	5,	6,	0,	1	
G,	F,	E,	D,	C,	B,	A,	G,	F,	E,	D,	C,	B,	A,	G,	F,	E,	D,	C,	B,	A,	G,	*&c.*
										G,	F,	E,	D,	C,	B,	A,	G,	F,	E,	D,	C,	*&c. &c.*
										1,	2,	3,	4,	5,	6,	0,	1,	2,	3,	4,	5	

In which, the Figures, or Indices, affixed to the Letters, expreſſing their Diſtances from A in a retrograde Order, and likewiſe the Remainders from which the Dominical Letters are to be found; it is plain, that to find any Dominical Letter, as C in the *Gregorian* Calendar, it comes to the ſame Thing, whether you firſt find F in the *Julian*

lian Calendar, and increaſing its Index by 3, get 5, the Index of C; or find the Index immediately by this.

RULE (for the *Gregorian Stile*)

1. *To the given Year of our Lord add* 1, *and dividing the Sum by* 28, *the Quotient will be the Number of Cycles from the Year before the Birth of Chriſt*; *and the Remainder will expreſs the Place of the given Year in the Cycle.*

2. *To this Remainder add its Integer* 4*th Part*; *that is, on account of the Leap-years, reckon every* 4 *it contains to be* 5, *ſo ſhall the Sum give the Number of Dominical Letters that have been uſed from the Beginning of this Cycle*; *and the Remainder will be the Index of the Dominical Letter.*

If the given Year is a *Biſſextile*, then the Letter found will ſerve before the Intercalation, and the next below it for the reſt of the Year.

And if you have no Uſe for the firſt Part of the Rule, you may uſe the given Year, as you were directed in the latter Part of the Rule to uſe the Remainder; that is, add its 4th Part to it, and again add Unity, it being indifferent with reſpect to any Dominical Letter in the current

Cycle, whether all the preceding Cycles remain, or are neglected.

EXAMPLE I.

If the given Year is 1783, whoſe 4th Part is 445, and the Sum with 1 added, is 2233; this divided by 7, the Remainder is 3, the Index of E the Dominical Letter ſought.

EXAMPLE II.

Let the given Year be the laſt of this Century, 1799; its fourth Part added, and the Sum increaſed by 1, gives 2249; which divided by 7, leaves the Remainder 2, the Index of F.

29. This Rule would be perpetual, was a Day to be intercalated every 4th Year, as in the *old Stile:* but the centenaries of Years not deviſible by 4, as 1800, 1900, &c. being, for the Reaſon given in § 21, common Years, *in the new Stile*; the Rule muſt be accommodated to this Part of the *Gregorian* Correction.

The Dominical Letter for 1799, juſt now found, was F, and therefore thoſe for 1800, in the Old Stile, ſhould be E, D; and that of 1801 C: but the Year 1800, being reckoned a common Year, can have but one Letter E to mark its *Sundays* throughout. And the Dominical Letter of 1801 will be D, whoſe Index is leſs by Unity than that of C. To find therefore an Index (or Remainder) leſs by Unity, than it would have been by the Rule for the preceeding Century, we need only omit the Unity we then added to make up the Dividend of Letters; by which Means the Head of the Letters, G, will ſtand at our Saviour's Birth.

For the like Reaſon, in the Year 1900, G muſt ſtand one Year lower; or, which will amount to the ſame (ſince to find the Dominical Letter, we are concerned only with Septenaries, or the Multiples of 7) 6 Years higher: That is, we are to add 6 to $\frac{5}{4}$ of the given Year, to make a Dividend for the Diviſor 7.

This Rule will now ſerve for two Centuries, becauſe the Year 2000 is, in the New Stile likewiſe a Leap Year. But in 2100, the Number 5 muſt be added inſtead of 6; in 2200, the Number 4 muſt take the Place of 5: In 2300, the Number 3 muſt be added, which will now ſerve for two Centuries, till in 2500 it be changed to 2; and ſo on to Perpetuity.

The Change of the *Dominical Letters*, in the ſeveral ſucceeding Centuries, may likewiſe be explained as follows. Imagine that no Days were ſuppreſſed till (for Inſtance, the Year 1800; then the Number of the Days to be cut out, would be 12; that is, the Right Hand Column (in § 5) would be raiſed one Place higher; and not B but C would ſtand oppoſite to E. In 1900, there would be 13 Days to be cut out; that is, D would anſwer to E: In the Year 2100, 14, or twice 7, being cut out, the Letters in both Columns would be the ſame. And from theſe Suppoſitions, the ſame Rule would reſult, as was before given.

SECTION VI.

Of the Motion of the Moon; and the different kinds of Months.

30. THE *Moon*, beſides her apparent Motion, ariſing from the Diurnal Rotation of the *Earth*, in common with the reſt of the heavenly Bodies, has her proper Motion round the *Earth*, from *Weſt* to *Eaſt*: Obſerve her Place with reſpect to ſome remarkable fix'd Star, ſeveral Nights ſucceſſively; and you ſhall find that ſhe has got Eaſtward of it nearly at the Rate of 13

Degrees every Day; till after 27 Days, 7 Hours, ſhe is returned to the ſame fix'd Star. And hence the Time of this Revolution is called a *Periodical Month.*

Suppoſe the *Sun* and *Moon* to have ſet out together from the ſame *fix'd Star*, or rather from the ſame Point of the *Ecliptic*; after a *Periodical Month*, the Moon will be returned to the ſame Point; but the Sun, having advanced above a 13th Part of his annual Courſe, in that Time, it will require above a Day and a half ere the *Moon* can overtake him: That is, from *New Moon* to *New Moon*; (or from one *Conjunction* to the next following) there will be about 29 Days and a half; or a *Synodical* Month: And the ſame 29½ Days, will nearly meaſure the Time from any *Phaſis*, or Aſpect, of the Moon to the ſame *Phaſis* of the next Month; that is, from *Oppoſition* (or *Full Moon*) to *Oppoſition*, from firſt or 3d *Quarter*, to *firſt* or 3d *Quarter*, &c.

And the *Moon*'s Motion being unequal, for ſeveral Reaſons that are aſſign'd by the Writers of *Phyſical Aſtronomy*; ſo that the true, or *Aſtronomical* Month, cannot be brought into popular Uſe, in the Computation of Time: Inſtead of it, we uſe a *mean* Synodical Month of 29½ Days;

(ſetting aſide for the preſent the *Minutes*, *Seconds*, &c. by which it exceeds this Quantity, and which ſhall be taken into the Account hereafter)

31. The juſt Quantity of a *mean* Synodical Month, is had by taking the Number of Days, Hours, &c. between two diſtant *Lunar* Eclipſes, and dividing them by the Number of *Lunations* from the one to the other. By this means, whatever Errors may have been committed in ſtating the Times of the *mean Oppoſitions*, being divided amongſt ſo many *Lunations*, will in one *Lunation*, or in any ſmall Number of them, become inſenſible. As if the Errors of the Obſervations amounted even to 30′, or half an Hour, and the Number of *Lunations* was 300; the Error of a *mean* Synodical Month would be but $\frac{1}{10}$ of a Minute, or 6″: And if the Number of Lunations was greater, that Error would be diminiſhed in the ſame Proportion.

By this, and other equivalent Methods, Aſtronomers have fix'd the Length of a *mean* Synodical Month at $29^{d}. 12^{h}. 44'. 3'' \frac{1}{20}$.

Only the three Particulars following may be Noted. 1. That, beſides the Diſtance of the Obſervations uſed, the Choice of them is, in another reſpect, not altogether indifferent: they ſhould, if poſſible, be ſuch as were made when the *Moon's Orbit* as to its Poſition, Inclination to the *Ecliptic*, &c. was, nearly at leaſt, the ſame in both. 2. Regard muſt be had to the Difference of *Longitude* of the two Places where the Eclipſes were obſerved. As if a Place lies 15° Eaſt or Weſt of the Meridian of *London*, which is the 24th Part of the whole Circle: That Place has *Noon*, 1 o'Clock, 2, 3, &c. *ſooner* or *later*, by an Hour, than the Inhabitants of *London*: and ſo in proportion for any Difference of Meridions that may happen; allowing always 4 *Minutes of Time for a Degree*. 3. If the Conſequences of this Determination of the *mean* Synodical Month are extended to a very diſtant Futurity, it is not to be preſumed that they can be ſo juſt as in ſmaller Periods of Time; becauſe by ſuch an Extenſion, the Errors of the Obſervation will be multiplied.

32. The *Calendar* Months (*January*, *February*, *March*, &c. tho' they derive their Number, and their *general* Name, from the *Moon*, do not correſpond to her Motion; the *New* and *Full* Moons wandering thro' all the Seaſons of the Year. To trace them in this variable Courſe; and to ſhew how, after certain Periods, they can be brought back to the

ſame *Calendar* Day, is the Subject of what follows.

SECTION VII.

Of the Lunar *and* Luniſolar *Year.*

33. THE *Synodical* Month being rated at 29½ Days, and vulgar Uſe not admitting the Fraction of a Day, in the Computation of Time; two of theſe Months are added together, to make the Sum of 59 Days; and theſe Days are again divided into two Months; one of 30 and the other of 29 Days, a *full*, or complete, Month; and a *hollow*, or deficient one.

Twelve ſuch Months, *complete* and *deficient* alternately, or 6 Times 59 Days, make 354 Days, or a *Lunar* Year; wanting 11¼ Days of a common Solar Year of 365¼ Days: ſuch is the *Turkiſh* Year; whoſe Head does of conſequence run back thro' all the Seaſons, in ſomewhat leſs than 32½ *Julian* Years.

Theſe 11$\frac{1}{4}$ Days, in 4 Years, make 45 Days; which the ancient *Greeks* added as an *Emboliſmic*, (or Inſititious) Month at the End of every 4th Lunar Year, conſiſting by that Means of 399 Days, and the Beginning of the 5th Year was reſtored nearly to its former Place; the *Olympic* Games were celebrated in the ſame Seaſon of the Year, the ſame Month, and nearly at the ſame time of the *Moon*; as the Oracle had ordered: and theſe *Tetraerides*, or Periods of four Years, were thence called *Olympiads*.

And hence the Intercalation in the *Roman* Calandar of *Numa Pompilius* was copied; only that he divided the 45 Days into 2 Months of 22 and 23 Days, which he inſerted alternately every two Years, between the *Terminalia* and *Regifugium*, *i. e.* between the 23 and 24 of *February*, by the Name *Mercedonius*.

34. But the *Tetraeris* was ſoon found not to correſpond with the Heavens, and was therefore doubled into an *Octaeteris*, a Period of 8 Years; which was only repeating the Error in proportion to the Time.

Eight Tropical Years contain --	$2921^{d}.\ 22^{h}.\ 31'.$
99 *Synodical* Months contain --	$2923^{d}.\ 12^{h}.\ 21'.$
Exceeding the former by ——	$1^{d}.\ 13^{h}.\ 50'.$

a little more than a Day and a half in 8 Years; and which would require, that in about 19 times 8, that is, in 152 Years, a Month ſhould be ſuppreſſed.

The *Enneadecaeteris*, a Period of 11 Years, was next employed.

But 11 Years containing —— 4017^{d}. 15^{h}. $58'$.
And 136 *Lunations*, no more than 4016^{d}. 3^{h}. $51'$.

1 12 7

there was ſtill the Difference of $1\frac{1}{2}$ Day, by which the Period of 136 Lunations fell ſhort of 11 Tropical Years; which in ſomewhat more than a Century and a half, would amount to an *Emboliſmic* Month extraordinary, to be added at the End of the Lunar Year.

35. At length, *Meton*, the *Athenian*, obſerving probably that in the former Cycles, (the *Octaeteris* and *Endecaeteris*) the Errors were of contrary Denominations, the one in Exceſs, and the other in Defect; joined them into one *Cycle* of 19 Years: which reſtores the Head of the *Lunar* to that of the *Solar* Year to a ſurpriſing Exactneſs.

For 19 *Tropical Years* being 6939^{d}. 14^{h}. $29'$. $25''$.
And 235 *Lunations* containing 6939. 16. 31. 56.

The Difference is only —— 2. 2. 31.

Which will amount to the Exceſs of a Day, in no leſs than 223,3 Years, that is to 3 Days in 670 Years.

36. The Effect of this Difference of $2^{h}. 2'. 31''.$ will be ſeen, when we come to treat of the *Golden Numbers:* In the mean Time, let us conſider how the *Metonic Cycle* was anciently applied to the *Julian* Calendar.

To avoid Fractions, and make the Difference of the *Solar* and *Lunar* Years to be conſtantly 11 Days, they took the Year of 365 Days. And reckoning a *full* Month of 30 Days, from *Jan.* 1. the Second *(deficient)* Month, of 29 Days, ended *Feb.* 28; the 3d Month, of 30 Days, ended *March* 30; and the 4th, of 29 Days, ended *April* 28; there remaining 2 Days to *May* 1: Theſe added to 28 Days of *May*, made the 5th Month of 30 Days, and ſo on, reckoning *full* and *deficient* Months alternately, in the *odd* and *even* Places, till the *Lunar* Year of 354 Days, or twelve *Moons* of $29\frac{1}{2}$ Days each, was completed *Decemb.* 20, 11 Days before *Jan.* 1, of the Second Year.

Then reckoning a Month of 30 Days from *Dec.* 20, which will end *Jan.* 19; after that a

Month of 29 Days, ending *Feb.* 17, &c. the 2d *Lunar* Year will be completed *Dec.* 9; leaving 22 Days to be taken into the firſt Month of the 3d *Lunar* Year.

This computed the ſame Way, its 9th Month will end *Sept.* 1, and the 10th on *Sept.* 30, if it was a Month of 29 Days only. But it being a *Rule* (for a Reaſon that will appear preſently) that when two *Lunations* terminate in the ſame *Calendar* Month, the Month which ſhould in Courſe have but 29 Days, is to be reckoned of 30; excepting only in the 19th Year of the Cycle; by this Means, the 10th Month will be made to end on *Oct.* 1. the 11th on *Oct.* 30, the 12th in *Nov.* 29, and the 13th on *Dec.* 28th.

So that the 3d *Lunar* Year, which began *Dec.* 9, and ended *Dec.* 28; contains 13 Months, ſix of 29 Days, and ſeven of 30; being in all 365 Days, with 19 Days, the Difference between 28 and 9, that is 384; exceeding by 30 the *common Lunar* Year of 354 Days,

The Month *extraordinary*, which began *Sept.* 1, and ended *Oct.* 1, is called an *Emboliſmic* Month; and the Year of the Cycle in which this happens, has the ſame *Name*.

Add the 3 Days that remained of *December* to 27, and the firſt Month of the 4th *Lunar* Year will fall on *Jan.* 27; and continuing the ſame Proceſs, you will find that the 3d, 6th, 9th, 11th, 14th, 17th, and 19th, are *Emboliſmic*, or intercalary Years; in which there are 13 *Conjunctions*, or New-Moons.

You will find likewiſe, that the 235th *Lunation* will end where the *Cycle* began 19 Years before;

For 19 Times 365 Days make —— 6935^{d}.
And 19 *common Lunar* Years of 354^{d}. are 6726^{d}.
Add, for 6 *full* Emboliſmic Months, and one that was left *Deficient*, to make up the Account. —— —— } 209

The Sum is 6935^{d}.

37. If a Table was conſtructed, ſuch as Monſieur *Blondel* gives, of 19 Times 12 *Calendar* Months; and the Day of every Month on which the *New-Moon* falls, was marked for every Year of the *Cycle*; then we might, by Inſpection only, find the Age of the *Moon* for any given Day, provided we knew the Year of the *Cycle*.

But there is a readier Way of doing the ſame Thing. For ſeeing the 1ſt *Lunar* Year falls ſhort of the 1ſt *Solar* Year by 11 Days, the End of the 2d will anticipate 22 Days; and the 3d three Times 11, or 33 Days. Of theſe, 30 were employed to make up the *Emboliſmic* Month that ended *Oct.* 1, and the remaining 3 ſhew the *Moon*'s Age at *New-Year*'s Day of the 4th Year. In like manner, ſuppoſing the Year given to be the 17th of the Cycle, 17 times 11, or 187, ſhews the whole Number of Days, by which 17 *common Lunar* Years fall ſhort of 17 *Solar*, of 365 Days each. Divide 187 by 30, the Quotient 6 ſhews that there have been 6 Months intercalated ſince the Beginning of the *Cycle*; and the Remainder 7 gives the *Moon*'s Age at the End of the 17th Solar Year.

38. By theſe Remainders, which are called *Epacts*, or *additional* Days, the *Moon*'s Age is eaſily found, provided the Year of the *Cycle* is known. For Example, the *Epact* of the 18th Year, being 7, wanting 23 of 30, ſhews that the *New* Moon of *January* of the 18th Year, will fall on *Jan.* 23; and, counting 29 Days forward, the 2d *New Moon* will be *Feb.* 21, and the *Moon*'s Age, *March* 1, will be the ſame as *Jan.* 1; which muſt always happen, becauſe *January* and *February* added, make juſt two *Lunations*, or 59 Days.

The *New Moon* of *March* will therefore fall on the ſame Day of the Month, as did that of *January:* And, in general, to find the *New Moons* of *April*, *May*, *June*, *&c.* it is only ſubtracting the *Moon*'s Age at the Beginning of the Month from 29, 30, 29, *&c.* alternately.

And here it is to be obſerved, that in all the Months after *March*, the *New Moons* will anticipate with reſpect to the *Calendar* Days, at the Rate of about one Day in a Month. *March* and *April* have 61 Days, more by two than two *Lunations*, which make but 59; *May* and *June* the ſame; *July* and *Auguſt* have indeed 62, which exceeds the *Lunations* by 3 Days; but *September* and *October* give again a Difference of 2; as do *November* and *December*. And all theſe *Differences*, or *monthly Epacts*, added together, make, at the End of the Year, the *annual Epact*, or Difference of 11 Days; which, for the Reaſon already given, is not increaſed in the Months of *January* and *February*.

Hence we ſee the Ground of the common Rule for finding the *Moon*'s Age: " *To add to the* " EPACT, *that is to the* MOON'S *Age, on the* 1*ſt* " *of* January, *the Day of the Month, and Unity for*

" every Month that has run from *March* inclu-
" ſive ; ſo ſhall the Sum (cutting off 30, if it
" exceeds that Number) be the Moon's Age,"

As if the Year of the *Cycle* was 18, the *Epact* 7, and the given Time *June* 25 ; add 4 (for the Number of Months including *March)* to 32 (the Sum of 7 and 25) the Amount is 36 ; from which rejecting 30, reſts 6, the *Moon*'s Age : For if the *Calendar* Months had been each equal to the *Lunar*, the Addition of 7 to 25, would have given 2, the Day of the *New Moon* ; but as the *New Moon* has anticipated 4 Days in the *Calendar* ; thoſe 4 Days muſt be added, to find her Age on the propoſed Day.

39. All this proceeds upon the Suppoſition, that the 6 Hours above 365 Days of the *Julian* Year are neglected, which in 19 Years would riſe to 4^{d} 18^{h} : It may be aſked therefore, " What
" Proviſion is made in the Conſtitution of this
" *Calendar*, for exhauſting ſo great a Difference?"
And, " How happens it that the Rules given
" ſhew the *Moon*'s Age, and have always ſhewn
" it, to a tolerable Degree of Exactneſs ?

To this it is anſwered, 1°. That although the Authors of the *Calendar* were obliged, in order to adapt the 235 Lunations to a Cycle of 19 Years, to intercalate 6 Lunations of 30 Days, and a 7th of 29, in the Manner already deſcribed; yet they could not but know, that in ſo doing they over-rated the *Lunations:* for of the 6 *Emboliſmic* Months, 3 ſhould have had only 29 Days, which makes an Exceſs of 3 Days, and the 7th has but 29^{d}, which is too little by 12 Hours. Take theſe from 3 Days, and there remain $2\frac{1}{2}$ Days, the Error in Exceſs, by which the *Lunations* are rated above their true Number of Days. Take $2\frac{1}{2}$ Days from $4^{d}\frac{3}{4}$, the Remainder $2\frac{1}{4}$ Days is the Difference by which the 235 *Calendar Lunations* fall ſhort of 19 *Julian* Years. And adding the ſame $2\frac{1}{2}$ Days to $4\frac{3}{4}^{d}$, the Sum $7\frac{1}{4}$ Days is the Difference by which 235 *Lunations* of $29\frac{1}{2}$ Days each, fall ſhort of 19 *Julian* Years.

2°. If therefore the *mean* Synodical Month was no more than $29\frac{1}{2}$ Days, and the Year is $365\frac{1}{4}$ Days, there would be at the End of the *Metonic* Cycle, an intolerable Anticipation of the *New Moon*; which in 4 of theſe *Cycles* (or the *Calippic* Period of 76 Years) would require the Intercalation of a *deficient* Month of 4 times $7\frac{1}{4}$, or 29 Days: And, by this Means, 4 times 235 *Lunations* of $29\frac{1}{2}$ Days each, with an *additional Lunation* of 29 Days, would exactly meaſure the 76 Years.

3°. But the Truth of the Caſe is, the *Metonic* Cycle wanted not this Correction of *Calippus*, nor that *this* again ſhould, by the Addition of 8 Years, be changed into the Cycle of 84: it wanted only that the *mean* Synodical Month ſhould be taken of its juſt Length, 29^{d}. 12^{h}, 44 $3''\frac{1}{20}$.

For multiplying theſe $44'$. $3''\frac{1}{20}$, by 235, the Number of *Lunations*, they make 7^{d}. 4^{h}. $30'$; which want only 1^{h}. $30'$ to make up the 7^{d}. 6^{h}. by which 235 *Lunations* of $29\frac{1}{2}$ Days fell ſhort of 19 *Julian* Years.

But if the *Julian* Year is diminiſhed to its true Quantity of 365^{d}. 5^{h}. $48'$. $55''$, which is done in the *reformed Calendar*; the 235 *true* Lunations will exceed 19 Years by 2^{h}. $2'$. $31''$; as was already ſhewn.

SECTION VIII.

Of the Golden Number; its Uſe in finding the Moon's Age: And its Variation in the reformed Calendar.

THE *Golden* Number of any Year expreſſes the Place of that Year in the *Metonic* Cycle; as VII being the *Golden* Number, ſignifies that the current Year is the 7th of the aſſumed *Cycle* of 19 Years; and XIX, that it is the laſt Year. They take their Name from having been marked in the *Calendars* with gilt Letters.

The Choice of a Year from which the original *Cycle* ſhould commence, was altogether arbitrary: but being once fixed, no other ought to be admitted into the *Calendar*: and altho' the *Head* of the *Cycle* will ſtand at different Days in different Ages; as we now have I affixed to *Jan.* 23, which formerly ſtood at *Jan.* 20: yet it is one and the ſame *Cycle* of *Solar* Years; which does not exactly fall in with the 235 *Lunations.*

40. Reckoning the Cycle backwards, its firſt Year will coincide with the Year that preceded the Birth of *Chriſt*, according to the vulgar Account. That is to ſay, If we would find the *Golden Number* for any Year propoſed, uſe the following

RULE.

To the given Year add Unity, divide by 19, *and neglecting the Quotient, which only ſhews the Number of Cycles that are paſt, the Remainder will be the Golden Number; or, if nothing remains, the Golden Number is* 19, *that is, the Year propoſed is the laſt of the Cycle.*

EXAMPLE.

To find the Golden Number of the Year 1751, dividing 1752 by 19, the Quotient is 91, and the Remainder 4 is the *Golden Number.*

41. Reckon 17 *Cycles* downwards from the Year before the vulgar *Æra* incluſive, and the firſt Year of the 18th *Cycle* will be A. D. 323.

In which Year the *New Moons* falling on *Jan.* 23, *Feb.* 21, *March* 23, *April* 21, *&c.* the Number I. was affixed to all thoſe Days; becauſe, as they imagined, whenever the *Golden Number* ſhould be I, that is, every 19th Year, the *New Moons* would fall on theſe ſame Days.

In like Manner the *New Moons* of the ſecond Year, (A. D. 324) falling on *Jan.* 12, *Feb.* 10, *March* 12, *April* 10, *&c.* The Number II. was placed over-againſt thoſe Days: And by proceeding in the ſame Way to the End of the *Cycle*, that is the Year 341 incluſive, at the Time of the Council of *Nice*, or ſoon after, the *Golden Numbers* were diſpoſed as they ſtand in our *Calendar.*

The Uſe of this Diſpoſition is very plain; if, for Inſtance, it was required to find on what Day of *September* the New Moon falls in the Year 1751, in the *Old Stile?* It being ſuppoſed that the *New Moons* of the ſame Years of the *Cycles* will fall on the ſame Days of the Months to Perpetuity; and 1751 being the 4th of the current *Cycle*, or the *Golden Number* being 4; I ſee in my *Calendar* that A. D. 326, the 4th of the *Cycle*, the *New Moon* was *Sept.* 13, and thence might conclude, that in the Year 1751, it will fall on the ſame Day.

And 5, which is, in courſe, the Golden Number of 1752, belonging to *Jan.* 9, *Feb.* 7, *March* 9, *&c.* the *New Moons* ſhould then happen.

42. The *Golden Numbers* being once fix'd in their proper Places, would have ſerv'd to ſhew the *Moon*'s Age for ever, had the *Julian* Year of 365 $\frac{1}{4}$ Days been equal to the *Tropical*, and if 235 *Lunations* had exactly correſponded to 19 Years.

But we have already ſeen, that neither of theſe Suppoſitions is true; and that after 19 *tropical Years completed*, the *Moon* will ſtill want 2^{h}. 2′. 31″ of finiſhing the 235 *Lunations*, amounting to 3 whole Days in 670 Years.

And therefore, one erroneous Suppoſition being already corrected, by the Rule for ſuppreſſing 3 Days in 400 *Years*, and the *Year* thereby reduced to its juſt Quantity of 365^{d}. 5^{h}. 48″. 55‴; if we would prevent the Effect of the other, we may uſe this

RULE.

Advance the Golden Numbers one Day forward in the Calendar, three times every 670 *Years*; i. e.

one *Day after* 200 *Years*, another *after* 250 *Years, and a* third *after* 220 *Years*; *and ſo on to Perpetuity.*

By which means, the *Number* that now belongs to *January* 23, for inſtance, will, after 670 Years, be transferred to *January* 26; and the like of the whole Syſtem of *golden Numbers.*

43. But let us firſt correct the Error which the Want of this Rule has already occaſioned. At the End of the Year 322, the *golden* Numbers, 'tis ſuppoſed, were rightly placed; add to 322 two Periods of 670 Years, *i. e.* 1340, the Sum is 1662; in which Year, the *golden Numbers* ought to have been advanced ſix Places. The Number 1, which ſtood at *Jan.* 23, *Feb.* 21, *&c.* ſhould have ſtood at *Jan.* 29, *Feb.* 27, *&c.* and ſo on throughout the whole *Calendar*; to continue for two Centuries, when they are to move one Place further; after two Centuries and a half, one Place; and again, a Place further, after 220 Years; that is, in the Year 2332.

Or, neglecting the two Years, as inconſiderable, we may take for the *Radix*, or Commencement of this *Correction*, the Year 1660.

EXAMPLE I.

To find the correct Full-Moon *of* September 1751.

Place 4, the *golden Number* of the Year, which now ſtands againſt the 13th Day, 6 Places forward to the 19th; and ſubtracting 11, the Remainder ſhews that the *New-Moon* falls on *Sept.* 8, *Old Stile.*

The moving of the Number 6 Places forward gives the Day of the Month according to the *New Stile*; becauſe, in the *Rule*, a Year is taken of its juſt Length; or, which comes to the ſame, it is ſuppoſed that the *Gregorian* Correction of ſuppreſſing 3 Days every four Centuries, was in uſe every ſince *A. D.* 323.

EXAMPLE II.

To find Eaſter-Sunday *of the Year* 1753, *in the* New Stile.

Eaſter-Day being always the "firſt *Sunday* after "the firſt Full-Moon which happens after (or "upon) the 21ſt of *March*, or (the Full-Moon "happening on a *Sunday*) the *Sunday* following."

Look in the Month of *March* for the golden Number of 1753, which is 6, and the Day of the Month is the 28th. Count forward 6 Places, and you have *April* 3d the Day of the *New-Moon*; to which adding 14, the Sum ſhews the *eccleſiaſtical* Full-Moon to be *April* 17, with the Letter B. But G being the *Sunday Letter* of 1753, (by the Rules in the former Part of this Letter) which is 5 Places forward from B, adding 5 to 17, *Eaſter-Day* falls upon *April* 22.

EXAMPLE III.

To find Eaſter-Day *of the Year* 2440.

The golden Number being 9, which ſtands againſt *March* 25 of the old *Calendar*; and there having been above one whole Revolution of 670 Years ſince 1660, this number muſt be advanced 6 and 3, that is, 9 Places, till it ſtand oppoſite to *April* 3; add 14, and the *Full-Moon* falls on *April* 17; from which, counting on to E, the *Sunday* Letter of this Year, after *February*, *Eaſter-Day* will fall on *April* 20.

In general, therefore, to convert the *old* Calendar into a *new* and *correct* one, which ſhall ſhew the *Change* and *Full* of the Moon within a Day of the Truth; we need only " move the *golden Num-* " *bers* 6 Places forward ;" and, to keep it correct in future Ages, " they muſt be moved ſtill for- " wards, at the rate of 3 Days every 670 Years, " after the Year 1660," in the Manner already ſhewn.

Or, if it ſhould be thought more convenient, they may be advanced 6 and 14, that is, 20 Days; and then they will ſhew the *Full-Moons* *.

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* The two Cauſes of the *golden Numbers* ſhifting their Places have always been ſeparately conſidered: one as producing a *Progreſs* (or *Pröemptoſis*) of a Day in 130 Years; and the other a *Regreſs (Metemptoſis)* of as much in 310,7 Years, in the *Julian Calendar*. But the Method here uſed is more ſimple, by ſuppoſing the *Gregorian* Stile to have been always in uſe; and the Concluſion is, either way, the ſame.

SECTION IX.

Of the Epacts.

44. THE *Epact* of any Year being the *Moon's Age at the Beginning of* January, (or *March*) as was already mentioned, it follows, that there may be as many different *Sets* of *Epacts* as there can be different Ages of the *Moon* at the Beginning of the firſt Year of the aſſumed *Cycle*.

Gold N°	Epacts, N. Stile.	Epacts, O. Stile.
1	0	11
2	11	22
3	22	3
4	3	14
5	14	25
6	25	6
7	6	17
8	17	28
9	28	9
10	9	20
11	20	1
12	1	12
13	12	23
14	23	4
15	4	15
16	15	26
17	26	7
18	7	18
19	18	0
1	29(0)	11
2	11	22
3	22	3
&c.	&c.	&c.

For a *Cycle* may be ſuch, that, beginning with the Year, the *Moon*'s Age ſhall be any Number of Days from 0 to 29. And ſeeing the *Epacts* of all the ſucceeding Years of the *Cycle* are had by "the continual Ad-
"dition of 11 to the Epact
"of the firſt Year, rejecting
"30 as oft as the Sum exceeds
"that Number;" there will

ariſe 30 *Sets* of *Epacts*, for the 30 Caſes of the *Moon*'s Age of 0, 1, 2, 3, *&c.* to 29 Days.

45. The ſimpleſt *Set* is when the *Epact* of the firſt Year is 0, and the reſt as in the ſecond Column of this *Table:* which happens, at preſent, to be the *Gregorian Epacts.*

In this Set it is to be obſerved, that the *Epact* of the firſt Year of the ſecond *Cycle* being 29, theſe Days make the deficient *Emboliſmic* Month of the 19th Year of the foregoing *Cycle*; and the *Epacts* return to be 0, 11, 22, *&c.*

46. The third Column contains the *Julian* Epacts, beginning from 11, that is, from the ſecond of the other *Set.* They are found by "dividing the Product of 11 times the *golden Number* by 30, and reſerving the Remainder;" and are, in this Age, converted into the *Epacts* of the *new Calendar*, "by ſubtracting 11 from each, borrowing 30 when needful."

47. One Cauſe of Miſtake in the Affair of the *Calendar* has been the not attending to the *Progreſs* of the *golden Numbers* in the Calendar, at the rate of *one* Day in 223,3 Years; without which, the preſent Poſition of theſe Numbers cannot be ac-

accounted for; nor the *Moon*'s Age found, by *Epacts* derived from them.

Thus in our *Calendar*, 3 ſtanding againſt *Jan.* 1, indicates a *New Moon*; and yet, if we uſe the general Rule of multiplying by 11, to find the *Epact*, we might conclude the *Moon* to be then 3 Days old. The Reaſon is, That the *Cycle*, whoſe Years are inſerted in the *Calendar*, is, with reſpect to the *Calendar* Days, different from that which we uſe in finding the *Golden Number*. As in this little Table:

17.	18.	19.	1.	2.	3.	4.	*Julian Cycle.*
7.	18.	0.	11.	22.	3.	14.	*Epacts* (common)
1.	2.	3.	4.	5.	6.	7.	*Calendar Cycle.*

and, to make them coincide, all the Numbers in the *Calendar* muſt be placed 3 Days higher: that is, 1 muſt ſtand where 4 now is, 2 in the Place of 5; 3 in the Place of 6, *&c.* in the *Julian* Calendar.

For, by the *Progreſs* above-mentioned, the *Numbers* of the *Metonic Cycle*, from its firſt Inſtitution to about the Time of the Council of *Nice*, were advanced 3 Days. The Artiſts, therefore, who compoſed the *Calendar*, with great Judgment, retaining the Numbers of the original *Cycle*, diſpoſed them ſo as to correſpond with the *Lunations* of their own Age. Juſt as, 630 Years hence, they muſt ſtand 3 Days forwarder than at preſent: But then the *Epacts*, which decreaſe at the ſame Rate as the *Golden Numbers* advance, can no longer ſhew the Moon's Age, unleſs that Decreaſe be taken into the Account.

48. Concerning the *Epacts*, ſeveral other Things might be ſaid, which I cannot at preſent enter upon; and my Letter has already exceeded its Bounds. Their Uſe, for finding the *Moon's* Age without *Tables* or a *Calendar*, was already ſhewn. Beſides this, as they are derived from the *Golden Numbers*, they may be ſubſtituted for them in the *Calendar*, as they are in the *Gregorian*. But, for my own Part, I cannot ſee any great Advantage in this Method. The *Epact* of a particular Year, mark'd with a different Character, as we mark the Dominical Letter in an Almanack, would ſhew the *Fnll* or *New* Moon as well as the *Golden Number* does. But when the *Epacts* come to be altered, as in the Courſe of Ages they neceſſarily muſt be, any Rule for their Alteration will be more complicated than that which is here given for the *Golden Numbers* to ſhift their Places 3 Times in 670 Years.

As for Objections againſt the *Alteration of the Stile*, I have heard of none that are worth minding: Only ſome People mutter, " That " *Eaſter-Day* might be fixed at the *Sunday* af- " ter the *Vernal Equinox*, and then we ſhould " get rid of all this Stuff about *Golden Num-* " *bers* and *Epacts*: And that the devout Cele- " bration of that Feaſt cannot now have any " Relation to the Moon". To ſuch I ſhall only anſwer, that I wiſh they were better inſtructed, and were more ſenſible of the Reverence due to the Decrees of an *Oecumenical* Council, and to the Example of primitive Antiquity. I am,

SIR,

Your moſt obedient humble Servant.

March 24,
1750-1.

P. S. For hiſtorical Points I refer you to Dr. *Prideaux*, and hope you will excuſe any Inaccuracies that may have eſcaped me in ſettling the *Theory*.

FINIS.